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Classification of solutions for gauge fields on group manifolds

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Abstract. We impose the factorisation condition on a field theory constructed on a (super) group manifold. For the bidimensional Euclidean and pseudo-Euclidean groups the pseudo-connection 1-forms verifying the factorisation condition are classified in diffeomorphically equivalent classes. For the bidimensional Euclidean group we show the existence of solutions not diffeomorphically equivalent to those proposed by Ne'eman and Regge.

1. Introduction

The (super) group manifold approach was developed by Ne'eman and Regge (1978) to construct supersymmetric gauge theories, gravity and supergravity, from a geometrical point of view. In this geometrical formalism the gauge fields (or Yang-Mills potentials) (Ne'eman and Regge 1978, D'Auria *et al* 1980, D'Auria and Fré 1982, Castellani *et al* 1983) are a set of pseudo-connection 1-forms μ^A defined on a (super) group manifold G, where the index A runs in the adjoint representation of G.

The ordinary spacetime of these theories is a submanifold of the manifold G, and the supersymmetric gauge transformation are diffeomorphism in the (super) group manifold G. In these theories it is also assumed that the group manifold G has a bosonic subgroup H and that the physical solutions for the pseudo-connections 1-forms μ^A are factorised on the pair (G, H).

In a previous work (Foussats *et al* 1986) we have obtained the pseudo-connections μ^A in terms of a connection defined on the tangent bundle of the (super) group manifold G.

The pseudo-curvatures of these theories are defined by

$$R^A \equiv \mathrm{d}\mu^A + \tfrac{1}{2}C^A_{.BC}\mu^B \wedge \mu^C \tag{1.1}$$

where $C^A_{.BC}$ are the (graded) structure constant of the Lie algebra of the (super) group G.

The tangent vectors T_A, dual of the 1-forms μ^A, satisfy

$$\mu^A(T_B) = \delta^A_{.B} \tag{1.2}$$

$$[T_A, T_B\} = (C^D_{.AB} + R^D_{.AB})T_D \tag{1.3}$$

where $R^D_{.AB} = R^D(T_A, T_B)$.

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An essential assumption for these theories (Ne'eman and Regge 1978a, b) is the factorisation condition

$$T_H \rfloor R^A = R^A_{.HB} = 0 \qquad A, B = 1, \ldots, \dim \mathrm{G} \qquad H = 1, \ldots, \dim \mathrm{H} \tag{1.4}$$

where H is an index running in the adjoint representation of a bosonic gauge subgroup H of the (super) group manifold G.

Using local coordinates x^μ ($\mu = 1, \ldots, \dim \mathrm{G}$) for the (super) group G, the factorisation condition (1.4) is equivalent to the following equations for the tangent vectors T_A:

$$T^\rho_H(\partial_\rho T^\sigma_B) - T^\rho_B(\partial_\rho T^\sigma_H) = C^D_{.HB} T^\sigma_D \qquad B, D = 1, \ldots, \dim \mathrm{G} \qquad H = 1, \ldots, \dim \mathrm{H} \tag{1.5}$$

which can be deduced from equations (1.3) and (1.2).

Using the right action of the subgroup H on the (super) group G, it is possible to define the coset G/H. Choosing a representative element $\Lambda(z)$ for each z belonging to a subset U of the coset G/H, then for an element $g \in \mathrm{G}$ such that $g.\mathrm{H} \in U$, it is possible to write

$$g = \Lambda(z).\chi(g) \tag{1.6}$$

where

$$\chi : U \to \mathrm{H} \subset \mathrm{G}.$$

Using the map χ, Ne'eman and Regge (1978a, b) have obtained a solution for the factorisation condition (1.4), given by

$$\mu^A(g) = \chi^* \omega^A + [\mathrm{Ad}\, \chi^{-1}(g)]^A_{.B} \mu^B(z) \tag{1.7}$$

where ω^A are the left-invariant 1-forms of the (super) group manifold G, and $\mu^B(z)$ are arbitrary 1-forms defined on the coset G/H.

According to equation (1.7), the degrees of freedom remaining after using equation (1.4) are the fields $\mu^B(z)$ defined on the coset G/H. However, equation (1.7) is only a particular solution for equation (1.4).

Having in mind the functional quantisation of these theories, where the fields are to be considered in all the group manifold G rather than on the coset G/H, it is of interest to have the general solution of equation (1.4).

If the generating functional is assumed to be invariant under diffeomorphism on the (super) group manifold G, we would like to classify the 1-forms satisfying equation (1.4) in sets of diffeomorphically equivalent solutions. The classification will depend on the global structure of the (super) group manifold G which is considered.

In § 2, we perform the classification of the different factorised pseudo-connection 1-forms for the bidimensional Euclidean group E(2). For this case we prove that the different classes of solutions are classified by a non-vanishing integer number k and a rational number m/n. For $m \neq 0$, we obtain classes of solutions which are not diffeomorphically equivalent to those previously obtained by Ne'eman and Regge. Actually, the class of solutions obtained by these authors correspond to the value $m = 0$ in our classification.

In § 3, we give the classification for the case of the bidimensional Poincaré group, pointing out that there is only one equivalent class of solutions.

2. Classification of factorised solutions in the group E(2)

We we now going to consider the bidimensional Euclidean group manifold $G = \mathrm{E}(2)$, which is topologically equivalent to $R \times R \times S^1$. The subgroup H is in this case the group SO(2). A convenient representation for an element $g \in \mathrm{E}(2)$ is

$$g = \begin{pmatrix} \cos\varphi & \sin\varphi & x \\ -\sin\varphi & \cos\varphi & y \\ 0 & 0 & 1 \end{pmatrix} \tag{2.1}$$

where the parameters (x, y) run in $R \times R$ and φ is the parameter of the circle S^1.

Expanding $g^{-1}\,\mathrm{d}g$ in terms of the generators of the representation (2.1), it is possible to obtain the left invariant 1-forms defined on the group manifold E(2) and they are

$$\begin{aligned} \omega^1 &= \mathrm{d}x \cos\varphi - \mathrm{d}y \sin\varphi \\ \omega^2 &= \mathrm{d}x \sin\varphi + \mathrm{d}y \cos\varphi \\ \omega &= \omega^{(12)} = -\omega^{(21)} = \mathrm{d}\varphi. \end{aligned} \tag{2.2}$$

These left invariant 1-forms verifies the following Maurer–Cartan equations:

$$\mathrm{d}\omega^1 = -\omega \wedge \omega^2 \qquad \mathrm{d}\omega^2 = \omega \wedge \omega^1 \qquad \mathrm{d}\omega = 0. \tag{2.3}$$

Using equation (1.7) we can write the solutions of the factorisation condition proposed by Ne'eman and Regge (1978) for this case:

$$\begin{aligned} \mu^1 &= a(x, y) \cos\varphi - b(x, y) \sin\varphi \\ \mu^2 &= a(x, y) \sin\varphi + b(x, y) \cos\varphi \\ \mu &= \mathrm{d}\varphi + c(x, y) \end{aligned} \tag{2.4}$$

where $a(x, y)$, $b(x, y)$ and $c(x, y)$ are arbitrary 1-forms depending on the parameters (x, y) of the coset $\mathrm{G/H} = \mathrm{E}(2)/\mathrm{SO}(2)$. These arbitrary 1-forms are determined by the remaining field equations of the group manifold formalism (inner equations plus rehonomic conditions, see Castellani *et al* 1983).

Equations (1.5) in this case are

$$\begin{aligned} T^\mu \partial_\mu T_1^\rho - T_1^\mu \partial_\mu T^\rho &= -T_2^\rho \\ T^\mu \partial_\mu T_2^\rho - T_2^\mu \partial_\mu T^\rho &= T_1^\rho \end{aligned} \tag{2.5}$$

where T_1^ρ, T_2^ρ and T^ρ are the components of the tangent vector fields $\boldsymbol{T}_1$, $\boldsymbol{T}_2$ and $\boldsymbol{T}$, which are the duals of the 1-forms μ^1, μ^2 and μ respectively.

For each choice of vector field $\boldsymbol{T}$, it is possible to obtain the solution of equation (2.5). With this purpose, we define

$$\hat{T}_1^\rho \equiv \frac{1}{\sqrt{2}}(T_2^\rho - \mathrm{i}T_2^\rho) \qquad \hat{T}_2^\rho \equiv \frac{1}{\sqrt{2}}(T_2^\rho - \mathrm{i}T_1^\rho). \tag{2.6}$$

Replacing (2.6) in (2.5) we obtain

$$T^\mu(\partial_\mu \hat{T}_1^\rho) - \hat{T}_1^\mu(\partial_\mu T^\rho) = -\mathrm{i}\hat{T}_1^\rho \qquad T^\mu(\partial_\mu \hat{T}_2^\rho) - \hat{T}_2^\mu(\partial_\mu T^\rho) = \mathrm{i}\hat{T}_2^\rho. \tag{2.7}$$

The vector field $\boldsymbol{T}$ cannot vanish on E(2), because together with the vector fields $\boldsymbol{T}_1$ and $\boldsymbol{T}_2$ they give a basis for the tangent space to the group manifold E(2). Therefore it is possible to rewrite equation (2.7) using coordinates (x^1, x^2, x^3) such that

$$\boldsymbol{T} = \boldsymbol{\partial}/\partial x^3. \tag{2.8}$$

The general solution for (2.7) is

$$\hat{T}_1^{\rho} = f_1^{\rho}(x^1, x^2)\exp(-\mathrm{i}x^3) \qquad \hat{T}_2^{\rho} = f_2^{\rho}(x^1, x^2)\exp(\mathrm{i}x^3) \tag{2.9}$$

where f_1^{ρ} and f_2^{ρ} are arbitrary functions of x^1 and x^2.

The coordinates (x^1, x^2, x^3) are related to the coordinates (x, y, φ) defined at the beginning of this section by the invertible functions

$$x = x(x^1, x^2, x^3) \qquad y = y(x^1, x^2, x^3) \qquad \varphi = \varphi(x^1, x^2, x^3). \tag{2.10}$$

Fixing the coordinates x^1 and x^2 in (2.10), we obtain one integral curve of the vector field $\boldsymbol{T}$.

The vector fields (2.8) and (2.9) can now be written using coordinate components (x, y, φ):

$$T^x = \partial x/\partial x^3 \qquad T^y = \partial y/\partial x^3 \qquad T^{\varphi} = \partial\varphi/\partial x^3 \tag{2.11a}$$

$$\begin{aligned}
\hat{T}_1^x &= \left(\frac{\partial x}{\partial x^{\mu}} f_1^{\mu}\right)\exp(-\mathrm{i}x^3)\\
\hat{T}_1^y &= \left(\frac{\partial y}{\partial x^{\mu}} f_1^{\mu}\right)\exp(-\mathrm{i}x^3)\\
\hat{T}_1^{\varphi} &= \left(\frac{\partial \varphi}{\partial x^{\mu}} f_1^{\mu}\right)\exp(-\mathrm{i}x^3)
\end{aligned} \tag{2.11b}$$

$$\begin{aligned}
\hat{T}_2^x &= \left(\frac{\partial x}{\partial x^{\mu}} f_2^{\mu}\right)\exp(\mathrm{i}x^3)\\
\hat{T}_2^y &= \left(\frac{\partial y}{\partial x^{\mu}} f_2^{\mu}\right)\exp(\mathrm{i}x^3)\\
\hat{T}_2^{\varphi} &= \left(\frac{\partial \varphi}{\partial x^{\mu}} f_2^{\mu}\right)\exp(\mathrm{i}x^3).
\end{aligned} \tag{2.11c}$$

The field components in equations (2.11) must be periodic in the φ variable, and therefore it is necessary to verify

$$\begin{aligned}
x^1(x, y, \varphi + 2\pi) &= x^1(x, y, \varphi)\\
x^2(x, y, \varphi + 2\pi) &= x^2(x, y, \varphi)\\
x^3(x, y, \varphi + 2\pi) &= 2k\pi + x^3(x, y, \varphi)
\end{aligned} \tag{2.12}$$

for a non-vanishing integer number k.

The classification of the different solutions of equations (2.11) for the factorisation condition (1.4), is equivalent to the classification of the diffeomorphically equivalent vector field $\boldsymbol{T}$. This is a consequence of the invariance under diffeomorphisms $\Lambda: G \to G$ of the factorisation condition (1.4)

$$0 = \Lambda^*(\boldsymbol{T}_H \lrcorner R^A(\mu)) = \Lambda_*^{-1} T_H \lrcorner R^A(\Lambda^*\mu). \tag{2.13}$$

We start considering the possible integral lines corresponding to the tangent vector field $\boldsymbol{T}$ defined on the group manifold $\mathrm{E}(2) = \boldsymbol{R} \times \boldsymbol{R} \times S^1$. There are two different kinds of integral lines.

(1) Lines which do not wind around the circle S^1. This kind of line may be closed or opened. The closed lines should be excluded because they would give rise to a field $\boldsymbol{T}$ which vanishes in some point of the manifold.

(2) Lines winding n times around the circle S^1. This kind of line may also be closed or opened. If the lines are open, with an adequate diffeomorphism it is possible to transform the corresponding field into a constant component field pointing along one of the non-compact coordinates, say for example $\boldsymbol{T} = \boldsymbol{\partial}/\partial x$.

Therefore, it is not possible to verify the periodicity condition (2.12), and this kind of field must also be excluded.

It remains to consider a tangent vector field $\boldsymbol{T}$ with closed integral lines, winding n times around the circle S^1. For this case, the parametric equations (2.10) for the integral lines should have a period $L(x^1, x^2)$ in the parameter x^3:

$$\begin{aligned} x(x^1, x^2, x^3 + L(x^1, x^2)) &= x(x^1, x^2, x^3) \\ y(x^1, x^2, x^3 + L(x^1, x^2)) &= y(x^1, x^2, x^3) \\ \varphi(x^1, x^2, x^3 + L(x^1, x^2)) &= 2n\pi + \varphi(x^1, x^2, x^3). \end{aligned} \tag{2.14}$$

Using (2.12) and (2.14), we obtain

$$L = 2kn\pi. \tag{2.15}$$

Using coordinates (x, y, φ), a diffeomorphism $\Lambda: \mathrm{E}(2) \to \mathrm{E}(2)$ is given by three functions Λ^x, Λ^y and Λ^φ satisfying the periodicity conditions

$$\begin{aligned} \Lambda^x(x, y, \varphi + 2\pi) &= \Lambda^x(x, y, \varphi) \\ \Lambda^y(x, y, \varphi + 2\pi) &= \Lambda^y(x, y, \varphi) \\ \Lambda^\varphi(x, y, \varphi + 2\pi) &= 2\pi + \Lambda^\varphi(x, y, \varphi). \end{aligned} \tag{2.16}$$

Using (2.16) and (2.14) to transform the integral lines described by (2.10), we obtain new integral lines with the same integer numbers k and n.

Therefore, we have proved that integral lines, having different values of the integer number k or n, are not diffeomorphically equivalent.

There is also another number to be considered in the classification of the tangent vector field $\boldsymbol{T}$. For closed integral lines winding n times around the circle S^1, we can obtain a map of the plane $\varphi = 0$ on itself. Starting from a point $(x, y, \varphi = 0)$ we can move along the integral line passing through this point until we arrive at the intersection of this line with the plane $\varphi = 2\pi$. This intersection has an equivalent point in the plane $\varphi = 0$. In this way we obtain a map of the plane $\varphi = 0$ into itself. Calling M to this map, and taking into account that the integral lines close after winding n times around the circle S^1, we deduce that applying n times the map M is equivalent to a rotation of angle $2m\pi$ of the plane $\varphi = 0$, being m an integer number

$$(M)^n = R(2m\pi). \tag{2.17}$$

Integral lines having the numbers k, n and m defined by (2.12), (2.15) and (2.17) can be transformed by an adequate diffeomorphism into helical lines described by

$$\begin{aligned} x &= x^1 \cos\left(\frac{m}{n}\frac{x^3}{k} + x^2\right) \\ y &= x^1 \sin\left(\frac{m}{n}\frac{x^3}{k} + x^2\right) \\ \varphi &= \frac{x^3}{k}. \end{aligned} \tag{2.18}$$

The corresponding vector field $\boldsymbol{T}$ is

$$
\begin{aligned}
T^x &= \frac{\partial x}{\partial x^3} = -\frac{1}{k}\frac{m}{n}y \\
T^y &= \frac{\partial y}{\partial x^3} = \frac{1}{k}\frac{m}{n}x \\
T^\varphi &= \frac{\partial \varphi}{\partial x^3} = \frac{1}{k}.
\end{aligned}
\tag{2.19}
$$

For different values of the numbers k and m/n, (2.19) gives the components of vectors $\boldsymbol{T}$ which are the representative of different equivalence classes.

Using (2.6), (2.18) and (2.19) in equations (2.11) we can determine the (x, y, φ) components of the tangent vector fields $\boldsymbol{T}_1$, $\boldsymbol{T}_2$ and $\boldsymbol{T}$. Finally, it is possible to obtain the following expressions for the dual 1-forms μ^1, μ^2 and μ:

$$
\begin{aligned}
\mu^1 &= a(x, y, \varphi)\cos k\varphi - b(x, y, \varphi)\sin k\varphi \\
\mu^2 &= a(x, y, \varphi)\sin k\varphi + b(x, y, \varphi)\cos k\varphi \\
\mu &= k d\varphi + c(x, y, \varphi)
\end{aligned}
\tag{2.20}
$$

where a, b and c are 1-forms defined by

$$
\begin{aligned}
a &= a_1\left(r, \theta - \frac{m}{n}\varphi\right) \mathrm{d}r + a_2\left(r, \theta - \frac{m}{n}\varphi\right)\left(\mathrm{d}\theta - \frac{m}{n} d\varphi\right) \\
b &= b_1\left(r, \theta - \frac{m}{n}\varphi\right) \mathrm{d}r + b_2\left(r, \theta - \frac{m}{n}\varphi\right)\left(\mathrm{d}\theta - \frac{m}{n} d\varphi\right) \\
c &= c_1\left(r, \theta - \frac{m}{n}\varphi\right) \mathrm{d}r + c_2\left(r, \theta - \frac{m}{n}\varphi\right)\left(\mathrm{d}\theta - \frac{m}{n} d\varphi\right).
\end{aligned}
\tag{2.21}
$$

In the last expressions a_1, a_2, b_1, b_2, c_1 and c_2 are arbitrary functions and

$$
r = (x^2 + y^2)^{1/2} \qquad \theta = \tan^{-1}(y/x). \tag{2.22}
$$

We note that choosing $m = 0$, the pseudo-connections (2.20) are the 1-forms also obtained by using the expression (1.7) given by Ne'eman and Regge (1978a, b).

3. Classification of factorised solutions for the bidimensional Poincaré group

In this case, the group manifold G is the bidimensional Poincaré group, and it is topologically equivalent to $R \times R \times R$. The subgroup H is the Lorentz group SO(1, 1).

A representation for an element g of the Poincaré group is given by

$$
g = \begin{pmatrix} \cosh\eta & -\sinh\eta & t \\ -\sinh\eta & \cosh\eta & x \\ 0 & 0 & 1 \end{pmatrix} \tag{3.1}
$$

where the parameters t and x run in $R \times R$ and η is defined by

$$
\cosh\eta = (1 - \beta^2)^{-1/2} \qquad \sinh\eta = \beta(1-\beta^2)^{-1/2} \qquad \beta = v_x/c \qquad -c < v_x < 0. \tag{3.2}
$$

In this case, the left-invariant 1-forms are

$$\begin{aligned}\omega^0 &= -\mathrm{d}t \cosh\eta + \mathrm{d}x \sinh\eta \\ \omega^1 &= -\mathrm{d}t \sinh\eta + \mathrm{d}x \cosh\eta \\ \omega &= \omega^{01} = -\omega^{10} = -\mathrm{d}\eta.\end{aligned} \tag{3.3}$$

Using (1.7) we obtain a particular solution for the factorisation condition

$$\begin{aligned}\mu^0 &= a(t,x)\cosh\eta + b(t,x)\sinh\eta \\ \mu^1 &= a(t,x)\sinh\eta + b(t,x)\cosh\eta \\ \mu &= \mu^{01} = -\mathrm{d}\eta + c(t,x)\end{aligned} \tag{3.4}$$

where $a(t,x)$, $b(t,x)$ and $c(t,x)$ are arbitrary 1-forms depending on the parameters (t,x) of the coset G/H.

Equations (1.5) in this case are

$$T^\mu\partial_\mu T_0^\rho - T_0^\mu\partial_\mu T^\rho = -T_1^\rho \qquad T^\mu\partial_\mu T_1^\rho - T_1^\mu\partial_\mu T^\rho = -T_0^\rho \tag{3.5}$$

where T_0^ρ, T_1^ρ and T^ρ are the components of the tangent vector fields $\boldsymbol{T}_0$, $\boldsymbol{T}_1$ and $\boldsymbol{T}$, which are the duals of the 1-forms μ^0, μ^1 and μ respectively.

As in the previous section, for each choice of the vector field $\boldsymbol{T}$, it is possible to obtain the solution of equation (3.5).

We define the following new tangent vector fields:

$$\hat{\boldsymbol{T}}_0 = (1/\sqrt{2})(\boldsymbol{T}_0 + \boldsymbol{T}_1) \qquad \hat{\boldsymbol{T}}_1 = (1/\sqrt{2})(\boldsymbol{T}_0 - \boldsymbol{T}_1) \tag{3.6}$$

and then we choose coordinates (x^0, x^1, x^2) in such a way that the vector $\boldsymbol{T}$ becomes

$$\boldsymbol{T} = \boldsymbol{\partial}/\partial x^2. \tag{3.7}$$

Using (3.6) and the coordinate transformation which lead to (3.7), we obtain the following solution:

$$\hat{T}_0^\rho = f_0^\rho(x^0, x^1)\exp(-x^2) \qquad \hat{T}_1^\rho = f_1^\rho(x^0, x^1)\exp(x^2) \tag{3.8}$$

where f_0^ρ and f_1^ρ are arbitrary functions depending on x^0 and x^1.

Using the coordinates (t, x, η), related to the coordinates (x^0, x^1, x^2) by invertible functions, we also obtain

$$T^t = \partial t/\partial x^2 \qquad T^x = \partial x/\partial x^2 \qquad T^\eta = \partial\eta/\partial x^2 \tag{3.9a}$$

$$\begin{aligned}\hat{T}_0^t &= \left(\frac{\partial t}{\partial x^\mu} f_0^\mu\right)\exp(-x^2) \\ \hat{T}_0^x &= \left(\frac{\partial x}{\partial x^\mu} f_0^\mu\right)\exp(-x^2) \\ \hat{T}_0^\eta &= \left(\frac{\partial \eta}{\partial x^\mu} f_0^\mu\right)\exp(-x^2)\end{aligned} \tag{3.9b}$$

$$\begin{aligned}\hat{T}_1^t &= \left(\frac{\partial t}{\partial x^\mu} f_1^\mu\right)\exp(x^2) \\ \hat{T}_1^x &= \left(\frac{\partial x}{\partial x^\mu} f_1^\mu\right)\exp(x^2) \\ \hat{T}_1^\eta &= \left(\frac{\partial \eta}{\partial x^\mu} f_1^\mu\right)\exp(x^2).\end{aligned} \tag{3.9c}$$

In order to perform the classification of the possible non-equivalent solutions for the vector field T, we note that there is no periodicity conditions on the parameters (t, x, η) of the manifold, and therefore there is only one equivalence class of solutions. Choosing as a representative of this class the following vector field

$$T = \partial/\partial\eta \tag{3.10}$$

we have

$$x^0 = t \qquad x^1 = x \qquad x^2 = \eta$$

and (3.9) is

$$\begin{aligned}
&T^t = 0 && T^x = 0 && T^\eta = 1 \\
&\hat{T}^t_0 = f^0_0(t, x)\,\mathrm{e}^{-\eta} && \hat{T}^x_0 = f^1_0(t, x)\,\mathrm{e}^{-\eta} && \hat{T}^\eta_0 = f^2_0(t, x)\,\mathrm{e}^{-\eta} \\
&\hat{T}^t_1 = f^0_1(t, x)\,\mathrm{e}^{\eta} && \hat{T}^x_1 = f^1_1(t, x)\,\mathrm{e}^{\eta} && \hat{T}^\eta_1 = f^2_1(t, x)\,\mathrm{e}^{\eta}.
\end{aligned} \tag{3.11}$$

Using equation (3.6) we obtain the (t, x, η) components of the vector fields T_0 and T_1.

The corresponding pseudo-connection 1-forms are

$$\begin{aligned}
\mu^0 &= a(t, x) \cosh \eta + b(t, x) \sinh \eta \\
\mu^1 &= a(t, x) \sinh \eta + b(t, x) \cosh \eta \\
\mu &= \mu^{01} = -d\eta + c(t, x)
\end{aligned} \tag{3.12}$$

where $a(t, x)$, $b(t, x)$ and $c(t, x)$ are arbitrary 1-forms depending on the parameters (t, x), and they are determined by the remaining equations of the group manifold formalism.

4. Conclusions

It was suggested by Ne'eman and Regge (1978a, b) that if the pseudo-connection 1-forms are infinitesimally close to the factorised solutions (1.4), this solution should be diffeomorphic to (1.4). However, depending on the global properties of the group manifold, there could be other solutions for the factorisation condition.

For the group manifold E(2), we have proved that a non-vanishing integer number k and a rational number m/n classify the equivalence classes of solutions. For $m = 0$ the factorised solution proposed by Ne'eman and Regge is obtained. The existence of different equivalence classes is related to the periodicity conditions imposed by the compact parameter of the bosonic subgroup $\mathrm{H} = \mathrm{SO}(2)$ of the group manifold $G = \mathrm{E}(2)$.

For the bidimensional Poincaré group only one equivalence class of solutions is obtained. All the solutions for this case are diffeomorphic to the one obtained from the factorised solutions (1.4).

For the realistic four-dimensional Poincaré group there are compact parameters associated to rotations in the three-dimensional space. Therefore it is expected to be different equivalence classes. This work is in progress and will be the subject of a future publication.

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